Start of project

Upon starting the program, the following screen appears (make sure the macro security is set to medium to allow macros to be run).

Figure 1: starting screen

In figure 1 some of the terminology as used in the program is explained. The offset is defined as the horizontal distance between the two points the catenary shape is to form between. Sagging is defined as the tolerance for the Catenary to run below the preset depth. In other words, the lowest point of the Catenary is allowed to be located between the two end points. The program is started by pressing the start-button, (1.1).

Select output

After pressing start, the following input screen appears.

Figure 2: calculation method
A project name can be supplied in the textbox. Excel will automatically generate a worksheet named as the project name supplied here, followed by the number of the worksheet that is created.

An output variable has to be selected here. The program either calculates the required tension for a given anchor line length (Required Tension, 2.1) or the catenary shape at a given tension (Catenary Shape, 2.2).

**Input parameters**

![Figure 3: input parameters I](image1.png)

![Figure 4: input parameters II](image2.png)

After choosing which calculation method is required, one of the two screens as depicted above will show. These screens are placed on top of the output sheet. These screens show the input variables to be entered. Figure 3 represents the input screen for the calculation of the required tension, where figure 4 represents the catenary shape calculation.

The two screens only differ in that one asks for the anchor line length (3.2) while the other screen asks for the lateral force (4.4).

A short overview of the other input parameters is given below:

- **Water Depth (3.1)**
  The water depth can be calculated here. A depth larger than 0 needs to be entered. To approximate a ship to quayside anchoring, or a ship to ship anchoring from two points on the same height, a water depth of 0.1 can be selected where the allow sagging box needs to be ticked.

- **Anchor Line Type (3.2)**
  As given in figure 3 a number of predetermined anchor line types are included in the program. Depending upon the line type chosen, other input variables can be entered, picked from a list or will be calculated by the program.
  Should an anchor line type be required that is not present in the list, a user defined option is included as well.

- **Anchor Line Diameter (4.1)**
  Depending on the anchor line type that is selected a diameter can be entered or picked from a drop-down box.
- Anchor Line Density (4.2)
  For the predetermined anchor line types a density is given which cannot
  be altered. For the user-defined option a density has to be supplied.
- Ambient Density (4.3)
  The density of the medium (water/air) the anchor line is in can be entered
  here.
- Anchor Line Mass
  The anchor line mass is automatically displayed in mass per length.
- Horizontal Offset (3.4)
  The horizontal offset is the horizontal distance between two points in
  between which the Catenary should be formed. If the horizontal offset is
  not set, the program will generate a Catenary that ends horizontally at the
  defined depth.
- Allow Sagging (3.5)
  If you tick this box, the anchor line is allowed to sag.

To evaluate your project, press the calculate button. If you want to start a new
project, press the new project button, if you press the back button to achieve
this, your excel results of your last run will NOT be saved.
Output data

As mentioned before, the program generates a worksheet for each project. Such a worksheet provides all the data for and from the calculation, both numerically and graphically. The screen with numerical data is given below:

![Worksheet](image)

**Figure 5: numerical output**

The input parameters are given in the upper part. The relevant calculation results are printed in the middle part. Should the resulting stress be larger than the mean breaking load (MBL) of the anchor line, (only for ) a warning is given as well. In the bottom part the plot data for the graphical representation is stored.
The catenary shape is represented in a graph of which an example is given in figure 6.

![Graph](image)

**Figure 6: graphical output**

Special attention should be paid to the axes as they are not on the same scale. In some cases it therefore can look like the anchor line does not satisfy a catenary shape. One should therefore always check the scale of both axes.

**Constraints**

Within the program, some assumptions and simplifications have been made. These result in a list of constraints for the program which should be taken into account when performing calculations.

The following constraints are valid:

- The anchor line has no bending stiffness.
- No strain in the anchor line
- The anchor line diameter is constant over the anchor line length
- Homogeneous distribution of tension over the diameter of the anchor line
- When a steel chain is selected as anchor line type, the diameter entered represents the diameter of the shackle, not the diameter of the whole chain.
- The water depth has to be greater than zero. This also counts for a free-hanging cable in air. A mooring with zero vertical offset can be approximated by entering a 0.1 water depth.
- Anchor lines are not soaked.
- If the input results in a calculation of more than one million steps (for instance with a very steep sag), the program will terminate to avoid Excel to crash.
**Underlying theory**

**Introduction**

If a flexible chain or rope is loosely hung between two fixed points, it hangs in a curve that looks a little like a parabola, but in fact is not quite a parabola; it is a curve called a catenary, which is a word derived from the Latin catena, a chain. The word flexible points out that the rope or chain doesn’t have any stiffness and thus that the direction of the resulting force acting on any point of the chain is in the direction of the chain’s orientation at that point.

![Diagram of a catenary](image)

| $\psi$ | Angle between cable and horizontal at P |
| $\mu$ | Density per length |
| $s$ | Length of Cable |
| $T$ | Cable Tension |
| $T_0$ | Horizontal Tension at A |
| $g$ | Gravity |

**Figure 7: a catenary**

**The Intrinsic Equation to the Catenary**

The intrinsic equation describes the Catenary along the path of the Catenary $s$. We consider the equilibrium of the portion AP of the chain, A being the lowest point of the chain. See figure 1. It is in equilibrium under the action of three forces: The horizontal tension $T_0$ at A; the tension $T$ at P, which makes an angle $\psi$ with the horizontal; and the weight of the portion AP. If the mass per unit length of the chain is $\mu$ and the length of the portion AP is $s$, the weight is $\mu gs$.

From equilibrium we can simply see that:

\[- \quad T_0 = T \cos \psi \]

\[- \quad \mu sg = T \sin \psi \quad (1)\]

With Pythagoras and some basic geometry we can derive the next two expressions:

\[- \quad a^2 + b^2 = c^2 \]

\[- \quad (\mu sg)^2 + T_0^2 = T^2 \quad (2)\]
\[
\frac{\sin x}{\cos x} = \tan x
\]

\[\Rightarrow \tan \psi = \frac{\mu s g}{T_0} \quad (3)\]

Introduce a constant \( a \) having the dimensions of length. With \( a \) and equations (2) and (3) we can come to the intrinsic equation (i.e. the \( s, \psi \) equation).

\[a = \frac{T_0}{\mu g}\]

\[\tan \psi = \frac{\mu g s}{T_0}\]

\[\Rightarrow \tan \psi = \frac{s}{a}\]

\[\Rightarrow T_0 = \frac{\mu g s}{a} = \mu g a\]

\[(\mu s g)^2 + T_0^2 = T^2\]

\[\Rightarrow s^2 (\mu g)^2 + a^2 (\mu g)^2 = T^2\]

\[\Rightarrow T = \mu g \sqrt{s^2 + a^2} \quad (4)\]

**Equation of the Catenary in Rectangular Coordinates**

To get an expression for the Catenary in rectangular coordinates, we need to eliminate \( s \) from the equation.

Using equations (2) and (3) we can come to an expression for the derivative for \( ds/dx \) in terms of \( dx \) and \( dy \).

\[y' = \frac{dy}{dx} = \tan \psi = \frac{s}{a}\]

\[\Rightarrow \frac{d}{dx} a \cdot \frac{dy}{dx} = \frac{d s}{dx} = a \frac{d^2 y}{dx^2} \quad (5)\]
The Pythagorean relation between the intrinsic and rectangular coordinates for the Catenary can now be derived and is the second expression for the derivative for \( ds/dx \) in terms of \( dx \) and \( dy \).

\[
ds^2 = dy^2 + dx^2
\]

\[
\frac{ds^2}{dx^2} = \left(\frac{dy}{dx}\right)^2 + 1
\]

\[
- > \frac{ds}{dx} = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} = a \frac{d^2 y}{dx^2}
\]

Now that we have two equations (5) & (6) for \( ds/dx \) in terms of \( dx \) and \( dy \), we can eliminate \( s \) from the equation. Doing this gives a second order differential equation with boundary conditions. Solving this equation (defining \( y \) as a function of \( x \), so that it satisfies this equation and the condition \( y(0) = 0 \)) gives the Catenary equation in rectangular coordinates.

\[
y'' = \frac{1}{a} \sqrt{\left(y''\right)^2 + 1}
\]

Boundary Condition: \( y'(0) = 0 \)

\[
y' = \sinh \frac{x}{a}
\]

\[
y = a \cosh \frac{x}{a} + C
\]

Boundary Condition: \( y(0) = 0 \)

\[
- > y = a \cosh \frac{x}{a} - a, \quad a = \frac{T_0}{\mu g}
\]

Note that the equation can also easily be rewritten in terms of \( x \) and \( s \):

\[
\frac{s}{a} = \frac{dy}{dx}
\]

\[
\frac{dy}{dx} = \sinh(\frac{x}{a})
\]

\[
- > s = a \sinh(\frac{x}{a}), \quad a = \frac{T_0}{\mu g}
\]