Theoretical Simulation of the Measurement Process of Electrical Impedance Tomography

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1 ABSTRACT

Measuring the distribution of the density of slurry flow in a pipeline is a difficult task. This paper deals with the application of an electrical impedance tomography-based measurement technique for measuring this density distribution. The objective of the simulation presented in the paper is to lay a basis for the development of a measurement method that is capable of an on-line measurement of the concentration and velocity distribution of solids hydraulically transported in a pipeline.

The crucial part of the EIT instrument are; sensors, a data acquisition system (hardware) and an image reconstruction algorithm (software). The paper presents two software aspects that are important for a correct simulation of a physical quantity distribution from signals produced by the EIT-based measurement instrument. These aspects are the forward problem and the image-reconstruction problem.

A standard FEM package ANSYS was used for testing the FEM program that was specially developed for the research work. The iterative modified Newton-Raphson method (MNR) was used as image reconstruction algorithm, and a program based on this method was developed. The objectives of the image reconstruction simulations are, testing of the image reconstruction program and optimization of the parameters.

2 INTRODUCTION

Electrical Impedance Tomography (EIT) uses invasive, but non-intrusive electrical measurements to image materials based upon the differences in electrical conductivity. Compared with other tomography methods, EIT is inexpensive, poses few safety risks, and can operate under rugged conditions. Also, EIT can collect a full data set very quickly, making it valuable for the imaging of rapidly changing processes.

Application of current between the electrodes establishes a potential field that is a function of the unknown conductivity distribution. The electric field on the boundary is sampled by other flush-mounted electrodes. One such measurement is shown in figure 1, where the potential difference $\Delta V$, is measured between a pair of adjacent electrodes. Other measurements of potential difference can be made between all other pairs of electrodes on the boundary until all combinations have been made. The current drive pair of electrodes can then be switched, and more measurements can be collected. The process continues until a complete set of independent potential measurements has been generated.

After a complete set of data has been obtained, a numerical algorithm is used to invert the data to give an approximation to the conductivity distribution. The resulting image can then be related to the distribution of materials within the pipeline, such as the spatially varying particle volume concentration, or mixture flow densities.

Figure 1: The basic principle of measuring density or concentration profiles of a slurry flow in a pipeline using EIT.
3 EIT IMAGE RECONSTRUCTION

3.1 The governing equation and the boundary conditions:

If a conductor contains no electrical sources or sinks, the potential field inside of the conductor must satisfy
\[ \nabla \cdot (\sigma \nabla \Phi) = 0 \]  \hspace{1cm} (1)

In order to solve the equation, the boundary conditions have to be known. Two types of boundary conditions will exist: Dirichlet & Neumann types of boundary conditions
\[ \Phi = \Phi_i \quad i = 1...N, \]  \hspace{1cm} (2)

\[ \sigma \frac{\partial \Phi}{\partial n} = \begin{cases} +I & \text{On source electrode} \\ -I & \text{On sink electrode} \\ 0 & \text{Otherwise} \end{cases} \]  \hspace{1cm} (3)

3.2 Reconstruction algorithms

In order to reconstruct an image of the conductivity distribution, it is necessary to solve the equation \( \nabla \cdot (\sigma \nabla \Phi) = 0 \), and to obtain \( \sigma \).

Since \( \sigma \) is not uniform in the pipe cross section, usually, in EIT systems, the approach to solve the equation by using FEM—dividing the whole pipe cross section region into a finite number of regions, in each region, simply think the \( \sigma \) is uniform. A relation can be obtained between the voltage measurements made on the boundary and the conductivities of such regions. If there are \( N \) such regions then \( N \) simultaneous equations can be made to define the dependence of the conductivity values on the boundary measurements
\[ K \sigma = \Phi \]  \hspace{1cm} (4)

Where \( \sigma \) is a vector of conductivity values, \( \Phi \) is the vector of voltage measurements and \( K \) is the transformation matrix relating \( \sigma \) to \( \Phi \), if \( K \) and \( \sigma \) are known, this is easy to solve and is known as the “forward problem”.

For the real EIT situation, \( \Phi \) is known (measured) and \( \sigma \) has to be obtained.
\[ K^{-1} \Phi = \sigma \]  \hspace{1cm} (5)

This is known as the “inverse problem”.

3.3 Iterative Modified Newton-Raphson (MNR) image-reconstruction algorithm for EIT

A mathematical algorithm is needed to solve the inverse problem, in here the MNR method was used.

The basic steps of the MNR image reconstruction algorithm are as follows:

1. Divide the cross section into elements with conductivity \( \sigma \)
2. Reasonably guess the initial value \( \sigma_0 \)
3. Solve the forward problem, the \( n \)th boundary voltage \( \Phi_n \) due to the \( n \)th conductivity distribution \( \sigma_n \) is calculated using the FEM.
4. Compare the calculated \( \Phi_n \) with the measured \( \Phi^m_n \), if \( \| \Phi_n - \Phi^m_n \| > \varepsilon \), where \( \varepsilon \) is a prescribed error limit, then go to 5, otherwise stop.
5. Update \( \sigma_n \), \( \sigma_{n+1} = \sigma_n + \Delta \sigma_n \), where \( \Delta \sigma_n \) is dependent on the difference between the calculated and the measured voltages \( \Phi_n - \Phi^m_n \).
6. Repeat steps 3, 4 and 5 until a certain error limit \( \varepsilon \) has been reached.

4 SIMULATION & ANALYSIS

Both the program for solving the forward problem and the program for image reconstruction were developed by the author, and the former is indeed a FEM program based on variational finite element method using linear interpolation.

The theoretical simulations were carried out to test the software and optimize the parameters.

4.1 Forward problem simulation

A mathematical model is built in correspondence with the practical size of pipe, of which the inner diameter is \( d = 40mm \). The model is meshed with 176 elements and 105 nodes, and 104 conductivity elements are distributed in the 176 elements by a certain program.

Figure 2: Mathematical elements and conductivity elements distributing figure. (a) Mathematical elements, (b) Conductivity element distribution

In order to test the FEM program developed and also see whether the mathematical elements are sufficient for the requirement of the accuracy, a comparison with a standard FEM program is made; here ANSYS is used as the validation program. All other geometrical parameters are the same with the above; only the modeling has a fine mesh, 1513 nodes and 2896 elements. A uniform conductivity distribution is used and \( \sigma = 1mS/cm \), the injected current is 1mA.
The plot shows the potential difference between 13-paired sensors for $i^{th}$ current injection. From the plot we can see that the two lines match quite well. It means that the program works well and the elements used are sufficient. The program will further be used for solving inverse problem.

4.2 Image reconstruction simulation:

As described earlier, during the process of the MNR method, $\Delta \sigma$ must be known to update $\sigma_n$, $\sigma_{n+1} = \sigma_n + \Delta \sigma_n$. The least-squares method was used to find $\Delta \sigma$.

An object function $\mathcal{R}$ was defined as the inner product of the difference between $\Phi(\sigma)$ and $\Phi^m$:

$$\mathcal{R} = (\Phi(\sigma) - \Phi^m)^T W W (\Phi(\sigma) - \Phi^m)$$

To minimize $\mathcal{R}$, the following equation can be obtained:

$$[\Phi'(\sigma_i)]^T \Phi(\sigma_i) \Delta \sigma_i = -\Phi'(\sigma_i)^T [\Phi(\sigma_i) - \Phi^m]$$

(7)

solving the equation (7) will be a problem, since the condition of the Hessian matrix $[\Phi']^T \Phi'$ is very big. In mathematical fields, there are several methods being developed to solve an ill conditioned matrix. Here, the method used for the research, is the Levenberg-Marquardt method, simply known as the Marquardt method.

First let simply denote equation (7) as

$$Ax = b$$

(8)

Where A is the Hessian matrix

Let $\alpha_{i,j}$ be it’s $i^{th}$, $j^{th}$ element. The basic idea of the Marquardt method is modifying matrix A as

$$\alpha_{i,j}' = (1 + \lambda) \alpha_{i,j}$$

(9)

$$\alpha_{i,j}' = \alpha_{i,j} \ (i \neq j)$$

(10)

So the equation (8) changes to:

$$(A + \lambda I)x = b$$

(11)

Since the solution of the equation has a strong relation with $\lambda$, the value chosen is important.

The following simulations are carried out both for testing the process, and optimizing the parameter $\lambda$ to make the image reconstruction process convergent and with a fast speed.

For the first simulation, the process is according to the following steps:

**Step 1**

The initial conductivity values are given as: the conductivity of inner part of the model, $\sigma = 0.05 S/cm$, which includes 39 conductivity elements in correspondence with 64 mathematical modeled elements; the conductivity of the outer part of the model as: $\sigma = 0.1 S/cm$, which includes 65 conductivity elements in correspondence with 112 mathematical modeled elements. A full set of voltage data is calculated by using the FEM program. The data will be further used as ideal measured data to solve the inverse problem for the next step.

**Step 2**

The initial conductivity value is given as $\sigma = 0.01 S/cm$ in all elements. The process follows figure 4:

1. **Step 1:**
   - $\sigma_{\text{inner}} = 0.05$
   - $\sigma_{\text{outer}} = 0.01$
   - FEM
   - Voltage data
   - Least square object function
   - Update $\rho$

2. **Step 2:**
   - End

Figure 4: Image reconstruction simulation process

The image reconstruction program was used to reconstruct the image, with different $\lambda$’s taken. The initial condition of the Hessian Matrix is $3.08 \times 10^{-15}$ with $\lambda=0$, that is very high. The convergence of $\mathcal{R}$, which is defined by equation (6) is shown according to figure 5, with respect to different $\lambda$’s chosen.
For the first two curves, the $\lambda$ is not changed during the whole 20 times iteration. And for the bottom two curves, $\lambda$ is changed, with a factor, as follows:

The initial $\lambda = 1$, and

If $\mathcal{R}(\sigma_{k+1}) < \mathcal{R}(\sigma_k)$, $\lambda$ is decreased by a factor $\kappa$, otherwise $\mathcal{R}(\sigma_{k+1}) \geq \mathcal{R}(\sigma_k)$, $\lambda$ is increased by a factor $\kappa$.

The meaning of the value $\lg \mathcal{R} = \log_{10} \mathcal{R}$, in the situation, is that, if $\lg \mathcal{R} < -16$, the reconstruction accuracy of about 99.9% is reached.

From the above plot the factor $\kappa = 100$ is a much better choice, which has a low iterative time and a high convergence. Here an even higher order factor $\kappa$ was also tested, but the $\mathcal{R}$ was divergent.

Figure 6 shows the original and reconstructed image, the left side is the original image, and the right side is the reconstructed image in correspondence with it.

Since the order of the error in this case is very small, it is difficult to detect the difference between the original image and the reconstructed image by eye.

The above results are based on the initial estimated conductivity distribution close to the original conductivity values. If this is not the case, what will be the results? In order to get more information, a second simulation is carried as follows:

The steps are the same as above, the only thing is for the step 1, the forward step: the initial conductivity distribution is given as $\sigma = 0.02 \, S / cm$ for 10 conductivity elements of 104, which are distributed on 17 mathematical elements, and $\sigma = 0.1 \, S / cm$ for all other elements.

For the step 2, the initial estimated conductivity is given $\sigma = 0.2 \, S / cm$ for all elements.

Simulation was performed with different $\lambda$'s taken. The initial condition of the Hessian Matrix is $3.07 \times 10^{15}$.

Figure 6: The comparing figures between the original image and the reconstructed image

Figure 7: Convergence curve for different parameters

For the first two curves, the initial $\lambda = 1$ and the change factor $\kappa = 10$ and $\kappa = 100$ are given respectively. The change process is the same as it described in case one. The above figure shows that $\mathcal{R}$ is not convergent. That means if the initial guessed value is far away from the original value, there will be some problem for this kind of $\lambda$ changing. And indeed it is the flaw of the Marquardt method. In order to make it more tolerable, after a lot of tests, a better $\lambda$ changing rate chosen was found here both for convergence and for fast convergence.

It is described as follows:

Give an initial $\lambda = 1$,

If $\mathcal{R}(\sigma_{k+1}) < \mathcal{R}(\sigma_k)$, and

$$\frac{\mathcal{R}(\sigma_{k+1}) - \mathcal{R}(\sigma_k)}{\mathcal{R}(\sigma_k)} \geq \xi$$

$\lambda$ keep constant. After times testing, here a better $\xi$ chosen as 0.3 was found.

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If \( \mathcal{R}(\sigma_{k+1}) < \mathcal{R}(\sigma_k) \), and \( \frac{\mathcal{R}(\sigma_{k+1}) - \mathcal{R}(\sigma_k)}{\mathcal{R}(\sigma_k)} < \xi \): 
\[ \lambda \] decreases by a factor \( \kappa \).

If \( \mathcal{R}(\sigma_{k+1}) \geq \mathcal{R}(\sigma_k) \), \( \lambda \) increases by a factor \( \kappa \).

It is successful to reconstruct the image. The converge curves are the second two curves showed in the plot, remarked as \( \kappa^* = 10 \) and \( \kappa^* = 100 \).

The following figure is the original image (left side) and reconstructed image (right side) comparison,

Figure 8: The comparing figures between the original image and reconstruction image, the left is true image and the right is reconstruction image

5 MEASUREMENT RESULTS

The difference between the above simulation and really measurement is that the data used as measured voltages is free of error. And in the real test, the measured voltages have the error—which is unavoidable. So the distortion between the true image and reconstruction always exists. A real test was carried out in Dredging Lab, TU Delft. An unknown conductivity cobblestone was mounted inside the pipe which was filled with water. The water conductivity was measured in advance, \( \sigma_w = 0.1(S/m) \), (Resistivity \( \rho_w = 10\Omega m \)). The injected current is 1.5mA; the frequency of the current is 1000Hz.

Figure 9: A cobblestone hanged in the pipe

The reconstructed figure reflects the original image, the shape and conductivity of the image is similar with the cobblestone.

6 CONCLUSIONS

The results of the simulation presented in the paper show that if the measured data are error free and the first estimation of initial conductivity is good, the image can be constructed quickly and with high accuracy. However, if the sensed conductivity varies within a broad range (the maximum value 10 times higher than the minimum value) the Marquardt method finds it difficult to converge to the solution. Fortunately, this does not seem to be the case for sand-water mixtures. Although the difference in conductivity of sand and water is high, the conductivity of the mixture of 40% sand concentration is only twice that of water.

The optimization of the parameter is important for a quick convergence of the solution and for a creation of a sufficiently broad range of applicable conductivities.

The measurement results show that the designed instrument is capable of imaging the solid particle in water under simple modeled condition.

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