The Wedge Mechanism for Cutting of Water Saturated Sand at Large Cutting Angles

Dr. ir. S.A. Miedema2 & Ir. D.D.J. Frijters1

Abstract: The paper describes the results of the theoretical and experimental research of the cutting of water saturated sand at large cutting angles. Therefore cutting tests are carried out at blade angles of 60º, 75º and 90º, which are visually recorded. While the existing cutting theory of Miedema predicts infinite or negative cutting forces at large cutting angles, research is carried out to identify the failure mechanism while cutting at these angles. During these tests a wedge is observed at blade angles higher then 75º. This wedge is dynamic. Dynamics of the wedge consists of a stationary velocity profile within the wedge, which varies from a maximum velocity at the top side of the wedge, to a lower velocity along the blade surface. Blade angles between 45º and 75º form a transition region for the failure mechanism. Based on the cutting tests a theoretical model was constructed. This was implemented in a calculation model in Visual Basic, with which a prediction for the existence of a wedge can be given for various soils. With the theoretical model the velocity and deformation profile can be described.

Keywords: cutting of sand, kinematic wedge, soil failure mechanism

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**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Adhesion force</td>
<td>kN</td>
</tr>
<tr>
<td>b</td>
<td>Width of the blade</td>
<td>m</td>
</tr>
<tr>
<td>C2</td>
<td>Cohesion force along top side of wedge</td>
<td>kN</td>
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<tr>
<td>C3</td>
<td>Cohesion force along bottom side of wedge</td>
<td>kN</td>
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<td>e</td>
<td>Volume strain</td>
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<td>Fh</td>
<td>Horizontal cutting force</td>
<td>kN</td>
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<tr>
<td>Fv</td>
<td>Vertical cutting force</td>
<td>kN</td>
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<tr>
<td>g</td>
<td>Gravitation acceleration</td>
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<td>Hb</td>
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<td>m</td>
</tr>
<tr>
<td>Hc</td>
<td>Thickness layer cut</td>
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<tr>
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<td>Resulting force between wedge and quick layer cut</td>
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<td>K3</td>
<td>Resulting force on bottom side wedge</td>
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<td>Vc</td>
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<tr>
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<td>Relative velocity along blade</td>
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<td>Vt</td>
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<td>Force due to water-pressure on bottom side wedge</td>
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<td>W4</td>
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<td>Shear angle</td>
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<tr>
<td>δ</td>
<td>Friction angle sand/steel</td>
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<tr>
<td>ε</td>
<td>Angle between bottom side wedge and sand layer</td>
<td>degrees</td>
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<tr>
<td>θ</td>
<td>Wedge angle</td>
<td>degrees</td>
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<tr>
<td>λ</td>
<td>Friction angle between top side wedge and quick layer cut</td>
<td>degrees</td>
</tr>
<tr>
<td>ρw</td>
<td>Water density</td>
<td>kg/m(^3)</td>
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<tr>
<td>φ</td>
<td>Internal friction angle in sand</td>
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</tr>
<tr>
<td>χ</td>
<td>Angle between streaming resistance and cutting direction</td>
<td>degrees</td>
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1 INTRODUCTION

Considerations:
For cutting of water saturated sand at small cutting angles up to approximately 60° the cutting theory of Miedema (Miedema, 1987) forms a good approach of the cutting process. But the existing theoretical model predicts infinite or negative cutting forces for larger cutting angles. From cutting tests at these blade angles it is known that these forces remain limited. Based on those cutting tests, a different failure mechanism has been proposed for large cutting angles. The assumption of an alternative failure mechanism is based on a small quantity of picture material. It is described as a static wedge in front of the blade, which serves as a new virtual blade over which the sand flows away.

Starting points for research project:
As a general starting point the cutting theory of water saturated sand from Miedema will be taken. For the failure model, the existing finite-element calculations for pore pressures in a static wedge, as carried out by Mr. Ma (Ma, 2001), are used in this research. Moreover from several studies in soil mechanics a number of values for miscellaneous parameters are taken in this research. Additionally the Laboratory for Dredging Engineering with a cutting tank and a sand type is a starting point for this research.

Tasks during research project:
Firstly the cutting process at large cutting angles is identified by means of laboratory tests. These tests are carried out in the Laboratory of Dredging Engineering. Afterwards the observed failure mechanism was analysed. Then the failure mechanism of the sand layer was described mathematical and phenomenological. Finally a calculation model was developed with which the failure mechanism can be predicted.

2 FAILURE MECHANISM AND VELOCITY PROFILE IN DYNAMIC WEDGE

Based on the shear angle of the sand during the cutting process it can be determined where the sand enters the wedge and the layer cut. With use of continuity the start of the velocity profile will be derived. Also an indication of the complete deformation profile will be given. A dynamic wedge model with a set of characteristic angles, one of which is the shear angle, will be derived from the deformation profile.

For the cutting process the start of the velocity profile can be calculated with the continuity equation for the cutting of sand as in Figure 1. In the initial cutting layer the sand has a velocity equal to zero. Sand is cut at a velocity \( V_c \). Based on continuity all the sand cut has to be moved away, and carried away along the blade with a velocity \( V_r \). With this a pure continuity equation can be derived. The dilatancy takes place along a shear zone at angle \( \beta \), and the sand is carried away parallel to the blade. With these parameters the cutting velocity and the velocity relative to the blade can be expressed in one another.

\[
V_c \cdot H_i = V_r \cdot H_c \quad (1)
\]
\[
V_c \cdot l \sin \beta = V_r \cdot l \sin(\alpha + \beta) \quad (2)
\]
\[
V_r = V_c \frac{\sin \beta}{\sin(\alpha + \beta)} \quad (3)
\]

Figure 1: Continuity when cutting a sand layer

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The introduced blade angle $\alpha$ is only valid near the tip of the blade. Further on in front of the blade the sand will fail along the local virtual blade angle. With velocities known from (3) the global velocity of particles in the dilatancy zone can be determined. This is the sum of the vectors for the cutting velocity $V_c$ and the relative velocity $V_r$. These form the vector $V_t$, with which the total velocity can be described as seen from a global coordinate system. The resulting triangle as in Figure 2 will be called velocity triangle.

$$\begin{align*}
V_c & \quad \beta \\
V_t & \quad V_r
\end{align*}$$

Figure 2: Velocity triangle resulting from the continuity equation

Notice the angle between $V_r$ and $V_t$ doesn’t have to be 90°, because $V_r$ doesn’t have to be perpendicular to the dilatancy zone.

The velocity profile will vary along the failure curve. The blade angle will change from the blade tip in a virtual blade angle, or sand angle. Based on the continuity equation it is possible to compose the deformation profile within the dynamic wedge. The sand that comes into the wedge is not dilatated, until it’s in the wedge.

To analyse the cutting process and the occurring mechanisms, cutting tests with coloured sand were carried out. This coloured sand is placed in vertical grooves with a depth of 10cm ($\pm$ 1 cm), visible as vertical lines in Figure 3. As these lines are cut they will deform. In these tests it gets visible that the sands fails along a curve at the bottom of a wedge shape. This wedge shape is dynamic, where the dynamics exist of a stationary velocity profile. The failure curve starts at the blade tip and continues in a straight line further in front of the blade. The lines in the straight part of the failure curve will move on up with a maximum relative velocity $V_r$. The sand, which shears along this part of the failure curvature will be indicated as the quick layer cut. This quick layer cut seems to be moved up as a rigid body. Near the blade tip the coloured lines get curved. Near the blade the velocity is lower, what makes the failure lines get curved. The lower relative velocity near the blade is also due to the curved failure curvature.

The start of the velocity profile near the blade tip can be derived with the relation between the cutting velocity and relative velocity along the blade. After determination of the shear angle, based on minimum cutting force, the start of the velocity profile can be determined along the straight part of the failure curvature. The velocity profile is formed by the local magnitude of $V_r$.

As follows from the movies made during cutting test there is no exact boundary between the dynamic wedge and the quick layer cut. The friction in this area is not equal to the maximum internal friction angle in the sand in the initial layer cut. The effective friction angle is between $0^\circ$ and $\varphi$. The maximum friction is only locally exceeded. The angle of internal friction along the bottom of the quick layer cut is taken as a sharp straight line and is named $\lambda$. The friction along the blade is fully developed, resulting in movement along the blade. $\lambda$ doesn’t seem to be a soil parameter. In this research it is tried to calculate $\lambda$ with a calculation model.

Along the part of the failure curvature at the bottom of the quick layer cut, under an angle $\beta$, the angle of the deformation profile can be derived. When vertical coloured lines are placed in the sand layer the deformation can be followed, as was done during the cutting tests in this research. When a layer with the size of the dashed line is cut, as in Figure 4, the coloured lines will deform over this length. A point in the coloured line at the top of the layer cut will be cut earlier as a point deeper in the layer cut. A point which is cut earlier will start to move up with a relative velocity $V_r$, before a point deeper in the layer cut. At the top of the layer cut the angle $\theta$ can be determined. When the angle $\beta$ then is set as a constant, the angle at which the dashed line deforms can be determined. This deformation angle is valid along the total straight part of the failure curve.

Based on continuity the shaded triangle in the left illustration in Figure 4 must have the same size as in the right illustration as it is cut.
3 THEORETICAL WEDGE MODEL FOR CUTTING PROCESS AT LARGE CUTTING ANGLES

Based on the cutting test a theoretical model for the failure mechanism, with a wedge shape included, will be constructed. The wedge model consists of an initial sand layer, a layer cut and a wedge as in Figure 5. In this \( H_i \) is the height of the initial sand layer to be cut. \( H_b \) is the effective height of the blade. \( A \) is the original sand layer, the in situ sand layer. The layer cut can be split in 2 parts. These are the quick layer cut \( B \) and the dynamic wedge \( C \), from which the properties are investigated in this research. In the right illustration in Figure 5 the characteristic angles are drawn. In this illustration \( \alpha \) is the blade angle, \( \beta \) the shear angle or angle of the failure curvature. \( \theta \) is the wedge angle. The surface between the quick layer cut and the dynamic wedge will be indicated as the top surface of the dynamic wedge. In Figure 5 the wedge is indicated as a wedge with four sides. The length of the fourth side at the top surface of the wedge will be neglected based on the fact that a non negligible gives higher cutting forces as follows from the calculation model for the wedge. The bottom side of the dynamic wedge will, in contradiction to the deformation profile, be taken as a straight line. This as a simplification of the dynamic wedge model. The bottom side of the wedge is rotated with angle \( \varepsilon \). The bottom side of the wedge can’t be horizontally, because there is a velocity in the wedge, that can only follow from feeding the wedge with sand.
Dynamics of the wedge
The wedge is of the dynamic form. That means the sand flows continuously through the wedge, along a constant streamline pattern. The shape depends of braking out of the soil along the failure curvature based on the Mohr-Coulomb failure mechanism. The velocity of the particles can be derived with the continuity equation and be transformed in local and global velocities.

Accelerations in the wedge
The velocities in the wedge won’t be the same everywhere, because of the tapering shape of the wedge, and the friction along the sides of the wedge. Because of this there must be accelerations and decelerations within the dynamic wedge, for which energy must be available. This energy will contribute to the cutting forces. Because the size of the acceleration force is unknown, but doesn’t seem to be significant, it will be neglected in the rest of this research.

Mohr circle
Despite the dynamics of the wedge there will be a force and moment equilibrium around the wedge, because else the wedge would rotate or slide away. When we do the following extra assumptions we can describe the cutting process with the principle of the Mohr circle:

- There is an equilibrium for the resulting forces in any of two perpendicular directions
- There is an equilibrium of moments around any point
- The stress on a certain plane is constant
- This results in a point of action in the centre of the plane for the resulting force
- The Mohr circle describes the stress conditions in a point

Forces on the quick sand layer cut at large cutting angles

Figure 5: Wedge model with characteristic angles in the right illustration

Figure 6: Forces on the quick sand layer cut in the wedge model

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Forces in Figure 6 lead to the following force equilibriums around the quick layer cut.

**Horizontal force equilibrium:**
\[
K_1 \sin(\beta + \varphi) - W_1 \sin(\beta) + C_1 \cos(\beta) + T \cos(\beta) + G_2 \sin(\gamma) + W_6 \cos(\chi) \\
- C_2 \cos(\theta) + W_2 \sin(\theta) - K_2 \sin(\theta + \lambda) = 0
\] (4)

**Vertical force equilibrium:**
\[
-K_1 \cos(\beta + \varphi) + W_1 \cos(\beta) + C_1 \sin(\beta) + T \sin(\beta) + G_2 \cos(\gamma) + W_6 \sin(\chi) \\
+ C_2 \sin(\theta) + W_2 \cos(\theta) - K_2 \cos(\theta + \lambda) = 0
\] (5)

From these equilibriums $K_2$ can be found by eliminating $K_1$.

$K_2$ can now be derived and split in ($K_2 = K_{21} + K_{22}$):
\[
K_{21} = \frac{1}{\sin(\theta + \beta + \lambda + \varphi)}[W_2 \sin(\theta + \beta + \varphi) + W_1 \sin(\varphi) + W_6 \cos(\beta + \varphi - \chi)] \\
+ G_2 \sin(\varphi + \beta + \gamma) + T \cos(\varphi + C_1 \cos(\varphi - C_2 \cos(\theta + \beta + \varphi)]
\] (6)

\[
K_{22} = \frac{G_2 \sin(\beta + \varphi + \gamma) + T \cos(\varphi) + C_1 \cos(\varphi) - C_2 \cos(\theta + \beta + \varphi) + W_6 \cos(\beta + \varphi - \chi)}{\sin(\theta + \beta + \lambda + \varphi)}
\] (7)

The force $K_{21}$ is the part of $K_2$ in which the water-underpressures are given. In $K_{22}$ the gravitation force $G_2$, inertia force $T$, cohesion $C_1$ and $C_2$, and the streaming resistance $W_6$ are given. These equations for $K_2$ are almost the same as for modelling of small cutting angles; the difference comes from the quick layer cut, moving against the dynamic wedge, instead of against the blade.

The force $K$ is the resultant of the normal force $N$ and the shear force $S$, along the side with the index concerned. These are derived in the next equations.

\[
N = K \cos(\varphi), S = K \sin(\varphi)
\] (9)

**Forces on the wedge at large cutting angles**

Also around the wedge horizontal and vertical force equilibriums can be derived. Therefore the following figure can be given.

\[
\text{Figure 7: Forces on the wedge in wedge model}
\]

With neglecting of $C_3$, $C_4$, $A$ and $G_1$ remains:

**Horizontal equilibrium:**
\[
W_4 \sin(\alpha - K_4 \sin(\alpha + \delta) + K_3 \sin(\varphi + \varepsilon) - W_3 \sin(\varepsilon) - W_2 \sin(\theta) + K_2 \sin(\theta + \lambda) = 0
\] (10)
Vertical equilibrium:
\[ W_4 \cos \alpha - K_4 \cos(\alpha + \delta) - K_3 \cos(\varphi + \varepsilon) + W_3 \cos \varepsilon - W_2 \cos \theta + K_2 \cos(\theta + \lambda) = 0 \] (11)

With both equilibriums \( K_3 \) and \( K_4 \) are found, by eliminating \( K_4 \) and \( K_3 \) respectively.
By eliminating \( K_4 \), \( K_3 \) is found:
\[ K_3 = \frac{1}{\sin(\alpha + \delta + \varphi + \varepsilon)}(W_4 \sin(\delta) + W_3 \sin(\varepsilon + \alpha + \delta) + W_2 \sin(\theta - \alpha - \delta) + K_2 \sin(\alpha + \delta - \theta - \lambda)) \] (12)

In the same way by eliminating \( K_3 \):
\[ K_4 = \frac{1}{\sin(\alpha + \delta + \varphi + \varepsilon)}(W_4 \sin(\alpha + \varphi + \varepsilon) + W_3 \sin(\varphi) - W_2 \sin(\theta + \varphi + \varepsilon) + K_2 \sin(\theta + \lambda + \varphi + \varepsilon)) \] (13)

**Forces on the blade**

![Diagram showing forces](image.png)

Figure 8: Cutting forces on the blade in the wedge model

With neglecting \( A \) and \( W_5 \) the horizontal and vertical force equilibrium on the blade in the wedge model can be written:

Horizontal force equilibrium on the blade in the wedge model:
\[ F_h = -W_4 \sin \alpha + K_4 \sin(\alpha + \delta) \] (14)

Vertical force equilibrium on the blade in the wedge model:
\[ F_v = -W_4 \cos \alpha + K_4 \cos(\alpha + \delta) \] (15)

In order to turn the forces into stresses, required for a Mohr circle, the forces must be divided by the surface area on which the forces react.

**Neglections and assumptions in the wedge and wedge model**

**Neglections**
- Cohesion, which can be described as internal shear force.
- Adhesion, which can be described as external shear force.
- Gravitation force, which can be described as the force by the weight of the layer cut on the blade.
- Streaming resistance, which can be described as the resistance the cutting process experiences by the cutting process, that moves on due to continuous movement of the cutting process. The streaming resistance is proportional to the square of the cutting velocity, while the volume strain velocity is proportional to the cutting velocity. The square of the streaming velocity follows from the dependence of the kinetic energy.

**Assumptions**
- The top side of the wedge or left side of the quick layer cut is straight. The mobilized internal friction angle varies from \( \delta \) on the blade to \( \lambda \) at the upper side of the wedge. In between the friction angle varies between these two.
- All angles in the right illustration in Figure 9 are smaller than \( \phi \), or form a mobilised part of the internal friction angle of the sand. This gives the following equation:
\[
0 \leq \phi \leq \lambda \leq \delta \leq \phi
\]  
(16)

- The friction angle is a varying angle as seen from the blade, but is taken as a constant \( \lambda \) along the top side of the wedge. \( \lambda \) can’t be greater than \( \phi \), \( \phi \) is chosen as the maximum internal friction angle in the sand. For the internal shear angle in this text an angle between 30º and 45 º is taken as in (Verruijt, 1999), and the cohesion \( c = 0 \).

- The friction angle between steel and sand is formed from the angle between the friction force and normal force on the blade. From the measured results in Miedema (Miedema, 1987) it can be concluded that the friction angle varies between 24º and 35º with an average of 30º, which is taken for this research. From these measuring results there is no clear dependence of the soil stresses or blade angle observed with respect to the soil/steel friction angle.

- Because the sand in this research is cohesionless, there can’t be negative pulling forces, and \( N_3 \) must be pointed upwards. This means that:
\[
N_3 \geq 0 \text{ and } K_3 \geq 0
\]

4 CALCULATION MODEL FOR DYNAMICAL WEDGE

Analytical solution of the wedge problem

In this research eight unknown variables are taken into account in the calculation model. These unknown variables are \( \beta, \theta, \lambda, K_4, K_3, K_2, K_1, \varepsilon \). Besides the model contains a number of known variables, which describe the geometry. The soil parameters \( \delta \) and \( \phi \) are also known variables. When these variables are reduced to 6 by substituting a realistic value for \( \beta \) and \( \varepsilon \) an implicit solution for the model can be found.

\( K_2 \) and \( K_1 \) can be found with a force equilibrium around the cut soil layer as derived for small cutting angles.

Now only the unknown variables \( \theta, \lambda, K_4, K_3 \) remain.

There are only 3 equations for the solution of these unknown variables.

- Horizontal force equilibrium
- Vertical force equilibrium
- Moment equilibrium around the wedge

When \( \theta \) or \( \lambda \) are filled in as a constant the system of equations can be solved. As a verification this solution can be filled in the formulas for the cutting forces in the wedge model for large blade angles as in the model for small blade angles. The model with the smallest cutting forces will occur. From this paragraph it follows that the problem can only be solved implicitly.

Modelling of wedge model in Visual Basic to an implicit solution

To solve the wedge model with the Mohr circle the programming environment Visual Basic can be used. With this environment the problem can be handled mathematically and be combined with a simple graphical interface.
To convert the resulting forces in the wedge model into stresses, they are divided by the length of the planes on which they work. The Mohr circle describes the total stresses, or soil stresses. These consist of the sum of effective stresses and water-underpressures. From stress analysis it is known that the radius of the Mohr circle can be described in a formula:

\[
\left(\sigma_x - \sigma_M\right)^2 + \tau_x^2 = R^2
\]  \hspace{1cm} (17)

With:
- \(\sigma_M\) = Average stress in the centre of the Mohr circle
- \(\sigma_x\) = Principle stress in a random point \(x\)
- \(\tau_x\) = Shear stress in a random point \(x\)

When two points at the Mohr circle, for example \(x\) and \(y\), are known the center can be calculated and the complete Mohr circle can be described with use of the formula. With use of the lengths of the sides of the wedge, the normal and shear forces can be converted into stresses \(\sigma\) and \(\tau\). With normal and shear stresses a Mohr circle can be calculated in Visual Basic, in which the midpoint is based on the following formulas:

\[
\sigma_x^2 - 2\sigma_x\sigma_M + \sigma_M^2 + \tau_x^2 = R^2 \\
\sigma_y^2 - 2\sigma_y\sigma_M + \sigma_M^2 + \tau_y^2 = R^2
\]  \hspace{1cm} (18) \hspace{1cm} (19)

By subtraction the midpoint can be calculated. This gives:

\[
\sigma_M = \frac{\sigma_x^2 - \sigma_y^2 + \tau_x^2 - \tau_y^2}{2\sigma_x - 2\sigma_y}
\]  \hspace{1cm} (20)

With Visual Basic a Mohr circle can be calculated when the stresses in two points are known. An example can be seen in Figure 10. When the center of the Mohr circle is calculated for a triangle, the wedge shape, three centers can be calculated. These are indicated as principal stresses at the horizontal axis, with the sides with which the center is calculated in subscript.

- \(\sigma_{23}\) = Midpoint for the Mohr circle for stresses along top side (2) and bottom side (3) of the wedge
- \(\sigma_{24}\) = Midpoint for the Mohr circle for stresses along top side of the wedge (2) and blade (4)
- \(\sigma_{43}\) = Midpoint for the Mohr circle for stresses along the blade (4) and bottom side (3) of the wedge

For a description of the complete wedge the 3 described centers have to give a solution for the equation \(\sigma_{23} = \sigma_{24} = \sigma_{43}\) or an approximation of this equation. This equation has to be solved for the variables named above. From the K-values the normal and shear forces can be calculated. These can be converted into stresses and after subtracting of water-underpressures they become effective grain stresses. The lines under specific friction angles are valid for the total stresses. The horizontal connection lines represent the water-underpressures and connect the lines under the specific angles with the Mohr circle.

Figure 10: Mohr circle calculated in Visual Basic
Geometry for new wedge model

The geometry as used in the calculation model is given in Figure 11.

Figure 11: Model of cutting process, with L₁ to L₄ as used in the calculation model

With:

\[
L_1 = \frac{H_i - L_3 \cdot \sin \varepsilon}{\sin \beta}
\]

\[
L_2 = \frac{H_b - L_3 \cdot \sin \varepsilon}{\sin \theta}
\]

\[
L_3 = \frac{H_b}{\tan \theta} \cdot \tan \alpha - \left( \frac{H_b}{\tan \theta} - \frac{H_b}{\tan \alpha} \right) \cdot \tan \varepsilon \cdot \sin(\frac{\pi}{2} - \theta)
\]

\[
L_4 = \frac{H_b}{\sin \alpha}
\]

Data for calculation in Visual Basic model

For a calculation in the cutting process in Visual Basic the next configuration is chosen, which is also carried out during the testing program in this research.

\[
\alpha = 90^\circ
\]
\[
\beta = 20^\circ
\]
\[
\varepsilon = 5^\circ
\]
\[
\varphi = 42^\circ
\]

\[
p_{\text{real}} = \frac{\rho_w \cdot g \cdot V_c \cdot e \cdot h_i}{k_{\text{max}}} \cdot p_{\text{calculated}}
\]

\[
\rho_w = 1025 \, \text{[kg/m}^3\text{]} \\
g = 9,81 \, \text{[m/s}^2\text{]} \\
V_c = 0,2 \, \text{[m/s]} \\
e = 0,15 \, [\text{-}] \\
H_i = 0,07 \, [\text{m}] \\
k_{\text{max}} = 0,00034 \, [\text{m/s}] \\
p_{\text{calculated}} = 1 \, [\text{-}] \\
p_{\text{real}} = 62106 \, [\text{-}] \\
Cutting force = p_{\text{real}} \cdot b \\
b = 0,25 \, [\text{m}] \\
Cutting force = 15,526 \, [\text{kN}] 
\]
The used layer cut ratio \( H_b/H_i = 20/7 \) and \( k_i/k_{\text{max}} = 0.25 \).
The water-underpressures follow from (Ma, 2001).

**Procedure for the prediction of a wedge in the cutting process**

- Smallest \( \theta \) gives highest \( \lambda \), but lowest cutting forces.
- Now find the smallest \( \theta \) for all values of \( \delta \), for which the calculation model gives a solution
- Then find the combination of \( \delta \) and \( \theta \), with smallest cutting forces
- See if the cutting forces for this combination with wedge are smaller as for the combination without wedge.

Hereby the solution with smallest cutting forces will occur in practice.

- When a solution is found for which \( \delta \) is smaller as \( \delta_{\text{max}} \) (the friction angle steel / sand) and \( \phi \) is larger than \( \phi_{\text{max}} \), the wedge in theory is static. When the wedge is static it means \( \epsilon = 0 \) because the wedge than won’t be fed with sand, and for this geometry the calculation has to be done again. Then for the model without wedge the configuration with the smallest cutting forces has to be determined, because \( \delta \) in the model without wedge is maximum. Now the solution for the static wedge has to be compared with the solution with a dynamic wedge.

This procedure can be applied to the arbitrary examples in Figure 12 and Figure 13. In Figure 12 the \( \delta_{\text{max}} \) and \( \phi_{\text{max}} \) are indicated with a vertical and horizontal line respectively. As solutions for the wedge are found between these lines and the origin of the axes, the wedge is dynamic for these examples.

**Pre-conditions for the calculation model**

The accuracy with which the center of the Mohr circle will be determined can never exceed the step size with which the variable will be varied.

\( K_2 \) has to be positive else there will be no thrust force on the wedge, in other words the quick layer cut and the wedge have to push but not pull at each other.

The water-underpressures for a calculation are chosen from a predicted wedge shape and given tables from (Ma, 2001). Therefore a limited number of calculations could be carried out. Small shape variations are carried out with the same water-underpressures with neglecting variation of water-underpressures.

**5 CONCLUSIONS AND RECOMMENDATIONS**

**Conclusions**

The predicted failure mechanism of a sand layer at large cutting angles with a static wedge isn’t correct. A wedge shape does occur, but there is a velocity in front of the blade. This means the wedge is dynamical. The dynamics consists of a stationary deformation and velocity profile in the wedge which is visually recorded. Besides the velocity along the surface of the blade is visualised with a camera. The deformation process of the sand layer consists of a dynamic wedge over which a layer at high velocity flows away. Both the wedge and the quick layer are fed with sand from the initial sand layer. The failure curves of the wedge and quick layer cut lay continuously behind each other. The deformation curve at the bottom side of the wedge flows continuously into a straight line at the bottom side of the quick sand layer.

The cutting process can be described in a model. This model consists of an implicit set of equations. Therefore it isn’t possible to describe the wedge explicitly. Based on force and moment equilibriums around the dynamic wedge a calculation model in Visual Basic is constructed, with which the existence of a dynamic wedge can be shown. Another indication for the existence of a model with a dynamic wedge is that the cutting forces in a model with a wedge, is smaller than in a model without a wedge.

This research is done with a wedge model with a number of assumptions and pre-conditions. These consist of a number of soil parameters and pre-calculated water-underpressures. Besides the shape of the model is chosen with a straight top side and rotated straight bottom side. Also a number of neglections are done which are founded and or quantified.
Figure 12: $\lambda$ versus $\delta$ for combinations of $\alpha$, $\theta$ and $\epsilon$

Figure 13: Cutting forces versus $\delta$ for combinations of $\alpha$, $\theta$ and $\epsilon$
Summarising this research it can be concluded the theory of Miedema can be used for blade angles up to 45º. For blade angles between 45º and 75º the form of the cutting process can’t be defined. For blade angles of 75º and 90º it has been possible to show an alternative failure mechanism with a dynamic wedge. The exact point of arising of the dynamic wedge, based on the cutting process, doesn’t follow from this research.

Recommendations

It can be recommended to do tests with a continuously adjustable blade. With this it can be shown from which blade angle the dynamic wedge is formed and besides if the dynamic wedge disappears after decreasing this angle. After that this angle could be connected with the soil parameters.

Besides it could be recommended to connect the water-underpressures with the calculation model in Visual Basic. Each change of geometry in the model will include changes of water-underpressures. After coupling these models all possible solutions for the cutting process can be calculated with the model in Visual Basic.

6 REFERENCES


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