Discrete Element Method Modelling of Sand Cutting

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ABSTRACT

Discrete Element Modelling (DEM) is a method which is designed to simulate the behaviour of a material by modelling the interactions between discrete elements i.e. particles. The mechanical interactions between particles and also between particles and the walls in are modelled by springs, dash-pots and friction sliders. The properties of the material and interactions (Poisson's ratio, shear modulus and density, coefficients of restitution, rolling and static friction) relate to the particle properties and not to the bulk properties. If DEM (using software like EDEM™ & PFC™) is reliable by validation with analytical models, it will have many applications in dredging, specifically for investigating underwater cutting processes in sand, clay and rock. Since people have no real experience with this method in dredging, the first step is to model a number of well-known examples like passive earth pressure and the cutting of dry sand and compare the simulation results with the analytical solutions. During this first step the relation between particle and bulk properties has also been investigated. Since the EDEM™ software does not yet contain a code to calculate porosity changes and pore water pressure (without coupling with CFD), this is implemented during a second step. The final target of this study is to use the DEM software to possible modify Miedema and Evans analytical solutions for sand and rock cutting at 3000 m water depths, but in the future the modelling of erosion could also be a topic of research.

The current research consisted of becoming acquainted with the software and carrying out basic tests to verify the outcomes. During the research it emerged that the shape of the particles determines whether the expected behaviour of sand can be replicated. Also the relation between micro and macro properties, i.e. the static friction of particle and the angle of internal friction, was investigated. Using a particle consisting of a number of spheres with an irregular shape did give very good results. Passive earth pressure simulations gave a very good match with the analytical solution, as did cutting tests in dry sand with a number of cutting angles. Using a cutting angle of 90 degrees resulted in the occurrence of a wedge in front of the blade with a wedge angle corresponding to literature.

Keywords: DEM, shear angle, cutting, angle of repose, non-spherical particles

1 INTRODUCTION

The paper describes the theory of Discrete Element Method (DEM) and the application of EDEM™ to the field of the dredging engineering. This is done by verifying the model outcomes with analytical solutions for specific situation like Passive Earth Pressure, sand cutting etc. The main goal of this paper is to investigate the application possibilities of EDEM™ for dry and water saturated sand cutting. The Discrete

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Element Method is a way to simulate the mechanical response of systems composed of discrete elements i.e. particles. Generally, sand particle shapes are arbitrary: any particle may interact with any other particle; and there are no limits placed on particle displacement or rotation. The big issue when using DEM to model granular material such as sand is that of regulating the particle properties (micro properties) in simulation to establish the correct sand behaviour (macro properties, bulk properties). This will be extensively discussed in section 3. In the same section also the influence of the particle shape on the macro behaviour of sand specimens will be shown. To check the reliability of the simulation a number of simulations of sand tests such as passive earth pressure and sand cutting are simulated and the results of these simulations are compared with the results obtained from analytical methods.

After gaining an understanding of DEM principles and EDEM™ parameters, results of simulation of cutting of dry sand will be shown in section 4. After that the modelling of pore area variation per time interval for use in saturated sand, will be explained in section 5. The water stress/pressure inside the pores depends on the pore volume variation between two consecutive time intervals. This part will show how the pore area variation (2D) can be calculated between two consecutive time intervals. When modelling water stress/pressure inside the pores in EDEM™, this method can be used as a basic principle to get information from EDEM™ per time interval, to process the information and to feed back the processed information to EDEM™ before the next time interval.

2 DISCRETE ELEMENT METHOD & EDEM™

DEM is a group of numerical methods for computing the motion of a large number of particles like molecules or grains of sand. In 1971 Cundall applied DEM to rock mechanics to investigate the behaviour of rock particles. The original work of the DEM for granular materials for geomechanics and civil engineering application was reported in the series of papers by Cundall and Strack (1979a–d, 1982, 1983), which were based on earlier work by Cundall (1978) and Strack and Cundall (1978). The term ‘distinct element method’ was coined in Cundall and Strack (1979a) to define the particular discrete scheme that uses deformable contacts and an explicit, time domain solution for the equations of motion for circular and rigid particles.

DEM models the considered material as a collection of deformable discrete elements with their own translational and rotational degrees of freedom. A loading process can be given as a series of small load increments; and the state changing process is searched for in the form of a series of displacement increments. In the most widespread DEM versions the displacement increments are determined with the help of Newton’s Second Law of Motion.

The 1980s and 1990s saw rapid development and a broad spread of the DEM for granular materials. At that time a number of codes were developed to simulate granular material behaviour with particle systems formed by 2D discs of circular, elliptical or polygonal shapes or 3D solid particles of spherical, ellipsoidal or polyhedral shapes. EDEM™ is developed by DEM Solutions, which was founded by John Favier (2002) in England.

EDEM™ manages information about each individual particle (mass, temperature, velocity and so on) and the forces acting on it. It can also take into account the particle’s shape, rather than assuming that all particles are spherical. To calculate the motion of each particle, EDEM™ uses the Hertz-Mindlin (default) or Linear Spring contact model (user guide EDEM 2.1.1.). The inputs required for this software are particle properties such as:

- Density [kg/m³]
- Poisson ratio [-]
- Shear modulus [Pa]
- Coefficient of restitution [-]
- Coefficient of static friction [-]
- Coefficient of rolling friction [-]

Material properties of particle

Interaction properties of particle
Particle Diameter \([\text{m}]\)

EDEM\textsuperscript{TM} API allows using to code and implementing plugin contact physics, external forces and particle-generation. EDEM\textsuperscript{TM} is not 100\% open source software. This means to plug new modules in EDEM\textsuperscript{TM} the basic file of contact models of EDEM\textsuperscript{TM} should be used. In these file are the determination of the position of particles, contact definitions, factory of particles etc are included. The software used within this research is the Academic Version of EDEM\textsuperscript{TM} 2.1.1.

3 MODELLING OF PASSIVE EARTH PRESSURE & ANGLE OF REPOSE

3.1 GENERAL

In reality, the sand particles properties determine the bulk properties of sand. The diameter of the particles is almost the same and each particle has its own properties and unique shape. But, in the simulation the particle properties are exactly the same and the basic particles are spherical. The behaviour of the particle is determined mainly by two particle parameters, the coefficient of static friction and the coefficient of rolling friction. These two parameters have also direct influence on the bulk properties like angle of repose and the shear angle of the sand. This means that to get a certain angle of repose these parameters should be changed several times (iterative method). There are multiple combinations of these two parameters which give the sand specimen the same macro behaviour (e.g. the same angle of repose).

To simulate a sand sample with certain properties for this research we started to determine first the angle of repose by several sandpile simulations and then the angle of shear line by several Passive Earth Pressure simulations etc. The results of all simulation are compared one by one with analytical solutions.

All simulations in this research are repeated 5 times. Most of the simulations in this research are based on the same sand packing, which means that the sand specimen is already made by a simulation and it is saved and exported to use for other simulations. To investigate the influence of the sand dumping on the result of the simulations, also 5 simulations were done with different sand packing for the same test. The differences between the different simulations results are negligible.

3.2 SPHERICAL PARTICLES

The Passive Earth Pressure (PEP) is the pressure when a soil mass is externally forced to the limiting strength (i.e. failure) of the soil in compression Miedema (1984). It is the maximum lateral soil pressure that may possibly be exerted. At this maximum pressure, a triangular part of soil shears over other parts of soil. The required force just before shearing, the passive earth pressure force, and the angle of the shear line, known as the shear angle, can be calculated analytically. The passive earth pressure force and the shear angle can be calculated by means of the next equations Coulomb (1776):

\[
Q = \frac{1}{2} \cdot \gamma \cdot h^2 \cdot b \cdot K_p + 2 \cdot c \cdot h \cdot \sqrt{K_p} \tag{1}
\]

\[
K_p = \frac{1 + \sin \varphi}{1 - \sin \varphi} \tag{2}
\]

\[
\beta = \frac{\pi}{2} - \frac{\varphi}{2} \tag{3}
\]
Table 1 shows the sand properties and the dimensions of a certain PEP test. The passive earth pressure force and the shear angle is calculated on the basis of the given parameters (inputs) mentioned in this table.

![Figure 1: analytical modelling of Passive Earth pressure](image1)

### Table 1: Sand properties and other dimensions of a given PEP test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of internal friction</td>
<td>$\phi$</td>
<td>30</td>
<td>[°]</td>
</tr>
<tr>
<td>Height of the wall</td>
<td>$h$</td>
<td>0.034</td>
<td>[m]</td>
</tr>
<tr>
<td>Width of the wall</td>
<td>$b$</td>
<td>0.02</td>
<td>[m]</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c$</td>
<td>0</td>
<td>[kPa]</td>
</tr>
<tr>
<td>Coefficient of passive earth</td>
<td>$K_p$</td>
<td>3 [-]</td>
<td></td>
</tr>
<tr>
<td>Soil density (dry sand)</td>
<td>$\gamma$</td>
<td>16.32</td>
<td>[kN/m³]</td>
</tr>
<tr>
<td>Passive earth pressure force</td>
<td>$Q$</td>
<td>0.56</td>
<td>[N]</td>
</tr>
<tr>
<td>Shear angle</td>
<td>$\beta$</td>
<td>30</td>
<td>[°]</td>
</tr>
</tbody>
</table>

The shear angle and the passive earth pressure force can be determined by specific soil characteristics such as the angle of internal friction. Subsequently, the friction angle of the soil is determined by the type of soil particles. This means that the properties of the soil particles will determine the behaviour of the soil or, in other words, the soil particle properties will determine the soil properties. The soil particle properties are called micro properties and the soil properties are called macro (or bulk) properties.

![Figure 2: The possible motion of a particle in a PEP test after the occurrence of shear line](image2)

Usually the particle material properties such as density and particle diameter are virtually known for sand particles. These parameters can be taken over from the sand particle material properties. The particle

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interaction properties have the biggest influence on the motion of the particle. These parameters are not easy to determine in the case of sand particles, which means that these parameters should be determined on the basis of several simulations. After filling all the input parameters EDEM™ produces a certain number of particles (this is also an input parameter) with these properties. Figure 3 shows a simulated sand sample based on the chosen properties.

Table 2: The value of inputs in EDEM™ to simulated a sand sample

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle diameter</td>
<td>$D_{mic}$</td>
<td>2</td>
<td>[mm]</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$v_{mic}$</td>
<td>0.2</td>
<td>[-]</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>$G_{mic}$</td>
<td>$5 \times 10^7$</td>
<td>[Pa]</td>
</tr>
<tr>
<td>Density (specific gravity)</td>
<td>$\rho_{mic}$</td>
<td>2600</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>Coefficient of Restitution</td>
<td>$\psi_{mic}$</td>
<td>0.0001</td>
<td>[-]</td>
</tr>
<tr>
<td>Coefficient of Static friction</td>
<td>$\mu_{r,mic}$</td>
<td>0.57</td>
<td>[-]</td>
</tr>
<tr>
<td>Coefficient of Rolling friction</td>
<td>$\mu_{r,mic}$</td>
<td>0.008</td>
<td>[-]</td>
</tr>
<tr>
<td>Simulation time interval</td>
<td>$\Delta t$</td>
<td>0.01</td>
<td>[s]</td>
</tr>
<tr>
<td>Number of particles</td>
<td>$n$</td>
<td>3800</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Figure 3: A view of the particles and the box simulated by EDEM™ for PEP simulation, ($\gamma = 16.3$ kN/m$^3$)

By changing the particle interaction properties (micro properties) the properties of this sand sample also change. The properties of the sand sample, like for instance the angle of repose, can be regulated by changing the particle interaction properties. Figure 4 shows a sand pile simulation of the sand sample. The angle of repose of this sand sample is 30 degrees, which is obtained after several simulations and after changing the interaction particle properties, see Table 2.

Figure 4: Sandpile simulation by EDEM™

After this simulation, one of the macro properties, namely the angle of repose of the sand sample, is determined. To check the other bulk properties of the sand sample such as shear angle or the angle of
internal friction, a passive earth pressure is simulated. In this simulation a wall moves by a constant velocity of 0.01 m/s to the right. This wall does not have any friction with the box and the particles. Because the analytical solution is on basis on the 2D and the simulation is on basis on 3D, the static friction between the particles and the wall and the side walls of the box are negligible.

Figure 5: Passive Earth Pressure simulated by EDEM™

Figure 5 shows the simulation of the PEP test with the same sand sample, but obviously this simulation does not match the PEP sand test, because during the simulation no shear line occurs. After this simulation a lot of simulations with different particle interaction properties (for instance low static friction and high rolling friction) were run, but it was virtually impossible to regulate the sand bulk properties on the basis of the particle properties. This is because in each different sand test, sand has a different failure mechanism. Sand consists of a lot of different particles of different shapes, diameter etc, but in a simulation the sand sample consists of particles with approximately the same properties and it would be very time consuming to simulate a sand sample with a lot of different particles. In addition it is questionable if this would result in an improved simulation.

Another problem with such simulations is that the behaviour of the particle depends on three interaction properties, which produce many combinations. To limit the combinations, some of the properties have to be negligible.

The coefficient of restitution is set to zero to simulate completely inelastic particles. The coefficient of rolling friction determines how easily one particle can rotate. This coefficient is most relevant when the particles are perfectly spherical, but the influence of this coefficient is much less in the case of non-spherical particles. By producing non-spherical particles in the simulation the coefficient of rolling friction becomes irrelevant due to the particle shape.

### 3.3 NON-SPHERICAL PARTICLES

For the next simulation, the shape of the particles looks like the shape of sand particle, see Figure 6; the coefficient of restitution is chosen equal to zero; the coefficient of the static friction was set equal to 0.57 (equivalent to 30°) and the other parameters are the same as the previous simulation. This particle is made from several small spherical particles with the same properties and diameter. The diameter of this particle is approximately 2 mm, which is the same as diameter of spherical particles used in the previous simulations.
Figure 6: a) Non-spherical particle used in EDEM™; b) a photograph of a sand grain

To check the angle of repose of this sand sample, a sandpile simulation was performed, see Figure 7. According to the simulation the angle of repose of this sand sample is approximately 30°, \( \varphi = \arctan \left( \frac{2.5}{4.2} \right) = 30^\circ \).

Figure 7: Sandpile simulation in EDEM™ by non-spherical particles (\( \gamma = 16.3 \text{ kN/m}^3 \))

The result of the PEP simulation with non-spherical particles is shown in Figure 8. To decrease the calculation time of simulations, the dimensions of the sand sample are decreased, see Table 3.

Table 3: the dimensions of the new sand sample and the density of dense sand and loose sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( l )</td>
<td>0.075</td>
<td>m</td>
</tr>
<tr>
<td>Height</td>
<td>( h )</td>
<td>0.015</td>
<td>m</td>
</tr>
<tr>
<td>Width</td>
<td>( b )</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Density (dry dense sand)</td>
<td>( \gamma_{mac} )</td>
<td>16.3</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Density (relative dry loose sand)</td>
<td>( \gamma_{mac} )</td>
<td>15.1</td>
<td>kN/m³</td>
</tr>
</tbody>
</table>

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Figure 8 shows that the PEP simulation with non-spherical particles does show a clear shear line as expected from the theory. This simulation also shows a shear angle between 30° and 34°, which is very close to the analytical result (\(\beta = \frac{\pi}{2} - \frac{\phi}{2} = 45° - 15° = 30°\)). It looks as though by imitating the shape of the sand particle the sand sample has so far shown correct behaviour.

According to the analytical solution (\(Q = \frac{1}{2} K h^2\)) the maximum PEP force on the blade (Q the horizontal force on the wall) for this test should be equal to 0.055 N. Figure 9 shows a graph of the PEP force during simulation, which is calculated by EDEM™. At the beginning of the PEP simulation the PEP force increased very fast but after a short time this force decreased rapidly. This can be explained according to the Rowe (1980) method used to establish dilatancy stress, see Figure 10. This is because the sand sample is a dense sand sample. To check this phenomenon the sand sample used in the next simulation was relative loose sand (\(\gamma = 15.1 \text{ kN/m}^3\)). The PEP force for this new sand sample is shown in Figure 11. By comparing this figure with Figure 9, it could be concluded that EDEM™ simulates the correct behaviour.
Figure 10: Difference between PEP force for dense sand and loose sand, described by the Rowe method.

Figure 11: PEP force calculated by EDEM™ for loose sand

Based on the PEP simulation so far done with non-spherical particles, EDEM™ seems to simulate correct behaviour and the calculated forces are also close to the analytical results. As expected, the PEP force (Q, the horizontal force on the blade) for loose sand is lower than for dense sand. To check more parameters sand cutting is simulated in the next section.
4 MODELLING THE CUTTING OF DRY SAND

4.1 Analytical solution

The analytical solution for the dry sand cutting force, Miedema (2009), is derived from the analytical solution of water saturated sand cutting, which is given as follows, Miedema (1986):

\[ F_h = K_2 \cdot \sin(\alpha + \delta) \]  
\[ F_v = K_2 \cdot \cos(\alpha + \delta) \]  
\[ K_2 = \frac{G \cdot \sin(\beta + \varphi) + I \cdot \cos(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \]  
\[ G = \rho_s \cdot g \cdot H_i \cdot W \cdot \frac{\sin(\alpha + \beta)}{\sin \beta} \left\{ \frac{(H_h + H_i \cdot \sin(\alpha))}{\sin(\alpha)} + \frac{H_i \cdot \cos(\alpha + \beta)}{2 \cdot \sin \beta} \right\} \]  
\[ I = \rho_s \cdot v_c^2 \cdot \frac{\sin(\alpha)}{\sin(\alpha + \beta)} \cdot H_i \cdot W \]  
\[ N_2 = K_2 \cdot \cos(\delta) \]  
\[ \beta = 75 - \frac{\alpha - \varphi - \delta}{3} - \frac{\varphi - \delta}{2} - \frac{\delta}{4} \]

The cutting force depends on two different forces: Inertial force and gravity force, Miedema (2009).
4.2 Simulation with cutting angle of 90°

To simulate the sand cutting test, only the height of the wall of the previous simulation, Figure 8 needs to be changed while all other input remains the same, Figure 13. The shear angle simulated by EDEM™ is close to the analytical solution, which is derived from the water saturated sand, equation 10, $\beta = 75 - 30 - 15 - 0 = 30°$. In this simulation there is no friction between the particles and the blade.

![Figure 13: simulation of sand cutting $\varphi = 30^0$ & $\delta = 0$ (t₀ = 76 s)](image)

To check the angle of repose in this simulation, the blade is stopped after few seconds (Figure 14). Approximately the same repose angle as in the previous simulations, $\varphi = 31^0$, is obtained. This bulk property is also correct simulated by EDEM™.

![Figure 14: The angle of repose after, the blade stop, $V_{blade} = 0$ m/s)](image)

Figure 15 shows the cutting force calculated by EDEM™ during this simulation. According to analytical calculation the normal force on the blade must be equal to 0.28N, which is much bigger than the normal force calculated by EDEM™.
The gravity force from the analytical solution is equal to 0.16 N. This is based on the rectangular shape of the shear layer behind the blade as shown in Figure 12, which is the basis for sand cutting of water saturated sand. Because the water stress inside the pores and the water pressure around the shear layer in water saturated sand, the density and the volume of the shear layer is higher than the density and volume of shear layer in dry sand cutting, see Figure 13. But, because the equations for dry sand cutting are derived from the equation of water saturated sand cutting, the analytical gravity force in dry sand cutting is overestimated. Figure 16 shows the difference of shear layer volume from analytical sketch and EDEM™ simulation. By this figure, the difference between the cutting forces from both methods can be partly clarified. Another difference between these forces could also be caused by the fact that the simulations are calculated for 3D situation and analytical is basis on 2D. There is needed to investigate this subject with more researches.

In the next simulation the influence of the variation of the angle of static friction is checked. Figure 17 shows the behaviour of sand during sand cutting when the soil/steel friction angle is 2/3 of the angle of internal friction. Based on the theory, if the soil/steel friction angle $\delta \geq 2/3 \cdot \phi$, a triangle wedge of sand will be formed behind the wall. This could be assumed as a steel blade that pushes up the rest of the sand. To check this phenomenon in the next simulation the soil/steel friction angle is fixed to 20°. Short after moving of the blade, a part of sand behind the blade will form a triangle sand wedge. If the blade moves further, the angle of slope of this triangle varies between $57^\circ < \alpha - \phi < 61^\circ$ (for this simulation). This value is very close to the value of analytical method $\alpha - \phi = 60^\circ$. Also the shear angle in this simulation is calculated close to the analytical solution, $\beta = 75 - 30 - 15 - 5 = 25^\circ$. 

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**Figure 17a-b:** Behaviour of sand during sand cutting when $\alpha = 90^\circ$, $\varphi = 30^\circ$, $\delta = 20^\circ$

### 4.3 Simulation with cutting angle of 45°

To simulate the sand cutting test, to check the dependency of the result of the cutting angle, only the angle of the blade is changed to 45° compared to the previous simulation (Figure 8). The result is shown in Figure 18.
Figure 18: Sand cutting simulated by EDEM\textsuperscript{TM}, blue ball has the min. velocity and the red ball has the max velocity

When Figure 12 is compared with Figure 18, the simulation shows that by moving of the blade a part of the sand (shear layer, green balls) shears over another part of the sand which looks like the same as the shear layer in Figure 12. This simulation is also run for sand with different properties and different cutting angles. The results derived from EDEM\textsuperscript{TM} and the analytical solution of the shear angle and the cutting force are collected in Table 4 and Table 5.

**Table 4:** Cutting force calculated for different inputs by means of the analytical method and EDEM\textsuperscript{TM}

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th></th>
<th>EDEM\textsuperscript{TM}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 30^\circ$</td>
<td>$\phi = 45^\circ$</td>
<td>$\phi = 30^\circ$</td>
</tr>
<tr>
<td>$\alpha = 90^\circ$</td>
<td>$\delta = 0^\circ$</td>
<td>0.28 N</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\delta = 20^\circ$</td>
<td>0.85 N</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 45^\circ$</td>
<td>$\delta = 0^\circ$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\delta = 20^\circ$</td>
<td>0.21 N</td>
<td>0.30 N</td>
</tr>
</tbody>
</table>

**Table 5:** Shear angle calculated for different inputs by means of the analytical method and EDEM\textsuperscript{TM}

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th></th>
<th>EDEM\textsuperscript{TM}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 30^\circ$</td>
<td>$\phi = 45^\circ$</td>
<td>$\phi = 30^\circ$</td>
</tr>
<tr>
<td>$\alpha = 90^\circ$</td>
<td>$\delta = 0^\circ$</td>
<td>30$^\circ$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\delta = 20^\circ$</td>
<td>25$^\circ$</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 45^\circ$</td>
<td>$\delta = 0^\circ$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\delta = 20^\circ$</td>
<td>40$^\circ$</td>
<td>35.5$^\circ$</td>
</tr>
</tbody>
</table>

The cutting force calculated with EDEM\textsuperscript{TM} deflects somewhat from the analytical results for different simulations although the shear angle shown in the simulations is very close to the analytical results. According to the analytical solutions the cutting force depends on the inertial force and the gravity force of
the shear layer. The cutting speed used in the previous simulations is very low, which means that the inertial force may be negligible with respect to the gravity force of the shear layer. This means that the cutting force in these simulations mainly depends on the gravity force of the shear layer.

As was already explained in paragraph 3.2, the gravity force in the analytical solution is based on a shear layer of cutting in water saturated sand. Therefore, it is unrealistic to compare the cutting force calculated by EDEM™ and the cutting force from analytical results.

However, to check the behaviour of the sand and the reliability of the EDEM™ results, the results are compared with each other. For instance, by decreasing the cutting angle the cutting force will decrease, according to analytical results. EDEM™ also calculated a lower cutting force for a lower cutting angle. According to the analytical results, the cutting force will increase if the angle of internal friction increases. EDEM™ also calculated a higher cutting force when the angle of internal friction increases. Based on these, the simulations and the comparisons of the results with the analytical results, it could be concluded that the results obtained from EDEM™ show the correct behaviour of dry sand with variation of various input parameters.

5 MODELLING OF PORE VOLUME VARIATION PER TIME INTERVAL

It is the water stresses in the pores that account for the main difference between dry sand and water saturated sand. These water stresses mainly influence the cutting force, which is caused by the water flow into the pores and out of the pores. When the pores volume increases, the water flows into the pores and when the volume of the pores decreases the water has to flow out of the pore. If the pore volume increases faster than the water can flow into the pore, then cavitation will occur as the cutting force will rapidly increase. During the sand cutting in the shear zone the pore volume will increase very fast, depending on the cutting speed, see Figure 19. After shearing, the pore volume decreases slightly, because of the water stress inside the pores.

Figure 19: Analytical sketch of pore variation before and after shearing during sand cutting, Miedema (1986) [11]

To be able to calculate the cutting force, the water stresses inside the pores should be calculated. This cutting force cannot be calculated with the standard EDEM™ contact model. But EDEM™ gives the possibility to interact with of models, so it makes it possible to add external forces to the particles. The water stress/pressure inside the pore depends on the pore volume variation. By increasing the pore volume the contact between the particles could be broken or the distance between the particles could be made larger, the water stress/pressure keeps the particles together and vice versa.

So, one of the ways to estimate the water stress/pressure inside the pores is by calculating the pore volume variation per time interval. EDEM™ calculated the position and the number of the contacts of each of the particles per certain time interval. The water stress/pressure inside the pore influences the position or the...
number of contacts between particles. So, to be able to add the water stress/pressure to the particles per time interval, the pore volume variation should be calculated per time interval too Kruyt (2006) & Mohammadi (2003).

![Polygon model described by Kruyt (2006)](image)

To calculate the pore volume variation it is necessary to also know the position and the number of the contacts between each particle. Kruyt (2006) wrote a MATLAB® program, which calculates the position and the number of contact of the particles in 2D according to the polygon principle see Figure 21.

**Figure 20** This principle is used in this paper to calculate the pore area. To simplify the calculation it is supposed that the particles are circular and have the same radius and that each particle only has contact with other particles and not with the boundary. For this method the area of each polygon and the percentage of the particles that are present in this polygon are calculated according to the next equations.

**Determination of the area of each polygon:**

\[
\begin{align*}
\text{1) } & \quad \frac{y_{n+1} + y_n}{2} \left( x_{n+1} - x_n \right) \\
\text{2) } & \quad \frac{y_{n+2} + y_{n+1}}{2} \left( x_{n+2} - x_{n+1} \right) \\
\text{3) } & \quad \text{...} \\
\text{4) } & \quad \frac{y_n + y_{n+x}}{2} \left( x_n - x_{n+x} \right)
\end{align*}
\]

**Gauss Method**

**Determination of the area of each particle inside in the polygon:**

\[
A_{\text{disk}} = \pi R^2 \\
A_{\text{part of disk inside polygon}} = \frac{\alpha_{\text{the angle of polygon}} \cdot R^2}{2\pi} - \frac{\alpha_{\text{the angle of polygon}} \cdot R^2}{2}
\]

After this step has been taken the area of the pore can be determined for a certain time. But to calculate the variation of the pore area, the pore area difference between two respective time intervals should be determined. Because the number of polygons and the position of the particle will change between the time intervals it could be difficult to compare the pore area variation between two time intervals based on the polygons. For this reason, the whole area where particles are included would be a grid. The number and the position of each cell is the same for every time interval. This means that the properties of each cell could easily be compared between two time intervals. To be able to use this method the properties of each polygon have to be linked to the cells that are in that polygon. Before the linking of the cells to the polygon, the density of each polygon may be determined as follows:
\[ \rho_{\text{polygon}} = \frac{A_{\text{total area of disk in polygon}}}{A_{\text{polygon}}} \]

**Figure 21:** Variations in polygon density thus clarifying the pore area variation between two time intervals

When the density of the polygon increases, the pore area will decrease and vice versa, see Figure 21. The last step is to select and link the cells to each polygon. To select which cell belongs to which polygon, each cell has to be defined according to a number and the position of each corner and the centre of each cell is therefore defined as follows:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
t_1 & t_2 \\
t_1 + a & t_2 \\
t_1 + a & t_2 + a \\
t_1 & t_2 + a
\end{bmatrix}
\]

Cells(i).xc = t1+a/2;
Cells(i).yc = t2+a/2;

If the centre of a cell lies in a polygon, it is assumed that 50% of that cell is located in that polygon and that the cell will take over the density of that polygon. This is just a rough assumption and one that is good enough to investigate the position of a cell in a polygon. There are methods that one can use to determine exactly what percent of a cell lies in a polygon but the calculation time required is high, which is not efficient if the number of cells increases.

After linking the cells to certain polygons, each cell must be compared with the same cell in the next time interval. The density of all cells from time interval \( t_0 \) will be saved. In the next time interval \( t_1 \) the density of each cell will be compared with the same cell from the previous time interval. If the density of the cell is increased, the cells can be filled with the red colour and if its density decreases the cell is filled with the blue colour. If the density of a cell is the same as in the previous time interval or if the cell is not in a polygon it will be filled with the white colour, see Figure 22.

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So far the pore variations per time interval can be determined and compared between two respective time intervals. To be able to add the water stress/pressure to the particles in EDEM™ this model must be modified for 3D applications, because EDEM™ simulates in 3D space. The translation of the model to a 3D model is very difficult, because in 2D space, the model uses closed loops (polygon) and in 3D space no closed loops are present. This should be investigated by conducting more study and research. But an alternative way to calculate the pore volume variation in 3D space would be by using a big cube instead of a grid. The whole box should be divided into a big cube; all cubes have the same volume and a different density. By moving the particles inside the cube the density of the cube can vary. From this variation we can identify the water over pressure or the water under pressure to the particles, see Figure 23.

Figure 23: Cube method proposal to determine water stress. a) 3D model of cube and particle inside the cube; b) a top view of the cube; c) top view of the cube and the moving direction of the particles within a certain time interval; d) by moving the particle the density of the cube can be changed.
The advantages of this method are: it is capable of being used for 3D simulation and the particles can have any shape. The disadvantages of this method are: it calculates the cube density variation and not the pore volume variation; it is a much less accurate method; the cube density can be conveyed between two time intervals while the pore volume inside the cube varies between two time intervals. If the density of the cube is decreased that does not mean that the pore volume inside the cube will also decrease. Additional research is required to investigate this subject. One of the options to explore is through the coupling of EDEM™ with CFD which might give the possibilities to calculate the pore water pressure as required.

6 CONCLUSIONS & RECOMMENDATIONS

In reality the sand/rock particles are not perfectly spherical; their behaviour is determined by their natural shape and material properties. This research showed that by imitating the sand particle shape in the particles used in simulation, the behaviour properties of particles become easier to regulate. Through this method the coefficient of the rolling friction of particles will be determined by their shape. This also shows that the angle of internal friction of the specimen is the same as the coefficient of static friction of a particle. The results of simulation with non-spherical particles were compared to the analytical results and they showed that the results obtained from non-spherical particles are significantly closer to the analytical results than the results of spherical particles.

On the basis of these comparisons, it can be concluded that the macro behaviour of sand in EDEM™ can be modelled fairly realistic. Comparison of the analytical and the numerical solution of the cutting force in dry sand is unrealistic because the analytical method is based on water saturated sand, resulting in a difference of the gravity force. The gravity force of the shear layer in the analytical method is based on the shape of the shear layer in the case of water-saturated sand, which is bigger than the shape of the shear layer for dry sand. Therefore, the equations for the analytical method should be modified for dry sand cutting.

To model the water stress inside the pores, the 2D model calculates the variation of the pore area per time interval. The pore area variation is decisive for the water stress inside the pores. The advantage of this model is that the pore area variation is easy to visualize and calculate per time interval; this makes it possible to research the water stress/pressure on sand particles. But the disadvantage of this model is that it is devised for 2D problems and it is very difficult to convert it to 3D. An alternative to this model would be to use big square cells instead of small grids. The particles will lie inside these cells and they will determine the density of the cell. With this method the density of the cell can be compared during two consecutive time intervals. By moving the particles into the cell or when they are going out of the cell, the density of the cell will change, but that does not mean that the pore area will also change. The advantage of this method is that it can easily be converted to a 3D model.

The coupling of DEM and CFD might be interesting to investigate the numerical results of water saturated sand.

LIST OF SYMBOLS

\begin{align*}
C & \quad \text{Cohesion} \\
C_\mu & \quad \text{Friction} \\
h & \quad \text{Height of sand specimen in EDEM™} [\text{mm}] \\
H_b & \quad \text{Height of blade} [\text{m}] \\
H_i & \quad \text{Cutting depth} [\text{m}] \\
I & \quad \text{Inertial force} [\text{N}] \\
G_{\text{mic}} & \quad \text{Shear Modulus of particle in EDEM™} [\text{Pa}] \\
\end{align*}
\[ G \] \text{Gravity force of shear layer} \quad \text{[N]}
\[ K_L \] \text{Horizontal cutting force} \quad \text{[N]}
\[ K_n \] \text{Normal stiffness between two particles} \quad \text{[N/m]}
\[ K_P \] \text{Coefficient of passive earth} \quad \text{[-]}
\[ l \] \text{Length of sand specimen in EDEM™} \quad \text{[mm]}
\[ N_2 \] \text{Normal force on the blade} \quad \text{[N]}
\[ v \] \text{Cutting speed} \quad \text{[m/s]}
\[ V_{\text{mic}} \] \text{Volume of sand specimen in EDEM™} \quad \text{[m}^3\text{]}\)
\[ V_{\text{shear \_layer}} \] \text{Volume of shear layer} \quad \text{[mm}^2\text{]}
\[ Q \] \text{Passive Earth Pressure} \quad \text{[N]}
\[ \alpha \] \text{Angle of blade} \quad \text{[°]}
\[ \beta \] \text{Angle of shear line} \quad \text{[°]}
\[ \delta \] \text{Angle of steel/soil friction} \quad \text{[°]}
\[ \gamma_{\text{mic}} \] \text{Density of sand specimen in EDEM™} \quad \text{[N/m}^3\text{]}
\[ \mu_{\text{mic}} \] \text{Coefficient of rolling friction of particle in EDEM™} \quad \text{[-]}
\[ \mu_{s,\text{mic}} \] \text{Coefficient of static friction of particle in EDEM™} \quad \text{[-]}
\[ \rho_{\text{mic}} \] \text{Density of particle in EDEM™} \quad \text{[kg/m}^3\text{]}
\[ \rho_{\text{polygon}} \] \text{Density of polygon} \quad \text{[%]}
\[ \nu_{\text{mic}} \] \text{Poisson ratio of particle in EDEM™} \quad \text{[-]}
\[ \phi \] \text{Angle of internal friction} \quad \text{[°]}
\[ \psi_{\text{mic}} \] \text{Coefficient of Restitution of particle in EDEM™} \quad \text{[-]}

\text{REFERENCE}


[4] EDEM™ 2.1.2 User Guide 1


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